FEDERAL PUBLIC SERVICE COMMISSION
COMPETITIVE EXAMINATION-2018
FOR RECRUITMENT TO POSTS IN BS-17
UNDER THE FEDERAL GOVERNMENT

Roll Number

PURE MATHEMATICS

TIME ALLOWED: THREE HOURS	MAXIMUM MARKS = 100

NOTE: (i) Attempt **FIVE** questions in all by selecting **TWO** Questions each from **SECTION-A&B** and **ONE** Question from **SECTION-C. ALL** questions carry **EQUAL** marks.
(ii) All the parts (if any) of each Question must be attempted at one place instead of at different places.
(iii) Candidate must write Q. No. in the Answer Book in accordance with Q. No. in the Q.Paper.
(iv) No Page/Space be left blank between the answers. All the blank pages of Answer Book must be crossed.
(v) Extra attempt of any question or any part of the attempted question will not be considered.
(vi) **Use of Calculator is allowed.**

SECTION-A

Q. 1. **(a)** Let H and K be normal subgroups of a group G. Show that HK is a normal subgroup of G. (10)

(b) Let H and K be normal subgroups of a group G such that $H \subseteq K$. Then show that (10) **(20)**

$$\frac{(G/H)}{(K/H)} \cong G/K$$

Q. 2. **(a)** Show that every finite integral domain is a field. (10)

(b) Consider the following linear system, (10) **(20)**

$$x + 2y + z = 3$$
$$ay + 5z = 10$$
$$2x + 7y + az = b$$

(i) Find the values of a for which the system has unique solution.

(ii) Find the values of the pair (a, b) for which the system has more than one solution.

Q. 3. **(a)** Find condition on a,b,c so that vector (a,b,c) in $\mathbb{R}^3$ belongs to (10)

$W = span\{u_1, u_2, u_3\}$ where $u_1 = (1,2,0)$, $u_2 = (-1,1,2)$, $u_3 = (3,0,-4)$.

(b) Let W_1 and W_2 be finite dimensional subspaces of a vector space V. Show that (10) **(20)**

$dim W_1 + dim W_2 = dim(W_1 \cap W_2) + dim(W_1 + W_2)$

SECTION-B

Q. 4. **(a)** Let $f(x) = \begin{cases} x^2 & if \ x \le 1 \\ x & if \ x > 1 \end{cases}$ (10)

Does the Mean Value Theorem hold for f on $\left[\frac{1}{2}, 2\right]$.

(b) Calculate the. $\lim_{x \to 0} \frac{ln sin 3x}{ln sin x}$ (10) **(20)**

Q. 5. **(a)** Evaluate $\int_{-1}^{5} |x-2|\,dx$. (10)

(b) Prove that $f_{xy}(0,0) \neq f_{yx}(0,0)$ if (10) **(20)**

$$f(x,y) = \begin{cases} x^2 y \sin\frac{1}{x} & when\ x, y\ are\ not\ both\ 0 \\ 0 & when\ x, y\ are\ both\ 0 \end{cases}$$

PURE MATHEMATICS

Q. 6. **(a)** Find the area of the region bounded by the cycloid $x = a(\theta - \sin\theta),\ y = a(1 - \cos\theta)$ and its base. (10)

(b) Find the equation of a plane through (5,-1,4) and perpendicular to each of the planes (10) **(20)**

$$x + y - 2z - 3 = 0 \text{ and } 2x - 3y + z = 0$$

SECTION-C

Q. 7. **(a)** Express $\cos^5\theta \sin^3\theta$ in a series of sines of multiples of θ. (10)

(b) Use Cauchy's Residue Theorem to evaluate the integral $\int_c \frac{5z-2}{Z(Z-1)} dz$ where C is the circle $|z| = 2$, described counter clock wise. (10) **(20)**

Q. 8. **(a)** Find the Laurent series that represent the function $f(z) = \frac{z+1}{z-1}$ in the domain $1 < |z| < \infty$. (10)

(b) Expand $f(x) = \sin x$ in a Fourier cosine series in the interval $0 \le x \le \pi$. (10) **(20)**

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