



**FEDERAL PUBLIC SERVICE COMMISSION
COMPETITIVE EXAMINATION-2016
FOR RECRUITMENT TO POSTS IN BS-17
UNDER THE FEDERAL GOVERNMENT**

Roll Number

PURE MATHEMATICS

TIME ALLOWED: THREE HOURS

MAXIMUM MARKS = 100

- NOTE:** (i) Attempt FIVE questions in all by selecting TWO Questions each from SECTION-A&B and ONE Question from SECTION-C. ALL questions carry EQUAL marks.
- (ii) All the parts (if any) of each Question must be attempted at one place instead of at different places.
- (iii) Candidate must write Q. No. in the Answer Book in accordance with Q. No. in the Q.Paper.
- (iv) No Page/Space be left blank between the answers. All the blank pages of Answer Book must be crossed.
- (v) Extra attempt of any question or any part of the attempted question will not be considered.
- (vi) **Use of Calculator is allowed.**

SECTION-A

- Q. 1.** (a) Prove that the normaliser of a subset of a group G is a Subgroup of G . (10)
- (b) Let A be a normal subgroup and B a subgroup of a group G . Then prove that $\langle A, B \rangle = AB$ (10) (20)
- Q. 2.** (a) Let a be a fixed point of a group G and consider the mapping $I_a : G \rightarrow G$ defined by $I_a(g) = aga^{-1}$ where $g \in G$. (10)
- Show that I_a is an automorphism of G . Also show that for $a, b \in G$, $I_a \cdot I_b = I_{ab}$ (10) (20)
- (b) Let $M_2(\mathbb{R}) = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} : a, b, c, d \in \mathbb{R} \right\}$ be the set of all 2×2 matrices with real entries. Show that $(M_2(\mathbb{R}), +, \cdot)$ forms a ring with identity. Is $(M_2(\mathbb{R}), +, \cdot)$ a field?
- Q. 3.** (a) Let $T: X \rightarrow Y$ be a linear transformation from a vector space X into a Vector space Y . Prove that Kernel of T is a subspace. (10)
- (b) Find the value of λ such that the system of equations (10) (20)
- $$\begin{aligned} x + \lambda y + 3z &= 0 \\ 4x + 3y + \lambda z &= 0 \\ 2x + y + 2z &= 0 \end{aligned}$$
- has non-trivial solution.

SECTION-B

- Q. 4.** (a) Using $\delta - \epsilon$ definition of continuity, prove that the function $\sin^2 x$ is continuous for all $x \in \mathbb{R}$. (10)
- (b) Find the asymptotes of the curve $(x^2 - y^2)(x + 2y) + 5(x^2 + y^2) + x + y = 0$ (10) (20)
- Q. 5.** (a) Prove that the maximum value of $\left(\frac{1}{x}\right)^x$ is $e^{1/e}$ (10)

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- Q. 6. (a) Find the area enclosed between the curves $y=x^3$ and $y=x$. (10)
- (b) A plane passes through a fixed point (a, b, c) and cuts the coordinate axes in A, B, C . Find the locus of the centre of the sphere OABC for different positions of the plane, O is the origin. (10) (20)

SECTION-C

- Q. 7. (a) Determine $R(z)$ where (10)
- $$P(z) = (z-z_1)(z-z_2)(z-z_3)(z-z_4) \text{ with } z_1 = e^{i\pi/4}, z_2 = \bar{z}_1, z_3 = -z_1 \text{ and } z_4 = -\bar{z}_1.$$
- (b) Find value of the integral $\int_c (z-z_0)^n dz$, (n any integer) along the circle C (10) (20)
.....with centre and z_0 radius r , described in the counter clock wise direction.
- Q. 8. (a) Use Cauchy Integral Formula to evaluate $\int_c \frac{\cos z + \sin 2z}{z - \pi/2} dz$ along the simple (10)
closed counter $C: |z|=3$ described in the positive direction.
- (b) State and prove Cauchy Residue Theorem. (10) (20)

