FEDERAL PUBLIC SERVICE COMMISSION
COMPETITIVE EXAMINATION-2024 FOR RECRUITMENT TO POSTS IN BS-17 UNDER THE FEDERAL GOVERNMENT

PURE MATHEMATICS

Roll Number

TIME ALLOWED: THREE HOURS	MAXIMUM MARKS = 100

NOTE: (i) Attempt **FIVE** questions in all by selecting **TWO** Questions each from **SECTION-A&B** and **ONE** Question from **SECTION-C. ALL** questions carry **EQUAL** marks.
(ii) All the parts (if any) of each Question must be attempted at one place instead of at different places.
(iii) Write Q. No. in the Answer Book in accordance with Q. No. in the Q.Paper.
(iv) No Page/Space be left blank between the answers. All the blank pages of Answer Book must be crossed.
(v) Extra attempt of any question or any part of the attempted question will not be considered.
(vi) **Use of Calculator is allowed.**

SECTION-A

Q. No.1.(a) Let N be a normal subgroup of a group G. If H is a subgroup of G, then prove that
$NH = \{nh : n\in N and h\in H\}$ is a subgroup of G. **(10)**

(b) If E is an epimorphism from a group G onto a group H then prove that G/K is isomorphic to H, where $K=KerE$. **(10)** **(20)**

Q. No.2. (a) Let R be a ring. If every $x\in R$ satisfies $x^2 = x$ then prove that R is a commutative. **(10)**

(b) For which value(s) of a will the following system have no solution? Exactly one solution? Infinitely many solutions? **(10)** **(20)**

$$x + 2y - 3z = 4$$
$$3x - y + 5z = 2$$
$$4x + y + (a^2 - 14)z = a + 2.$$

Q. No.3. (a) Determine a basis for and the dimension of the solution space of the system **(10)**

$$x - 2z + w = 0$$
$$3x + y - 5z = 0$$
$$x + 2y - 5w = 0.$$

(b) Let $v_1 = (1,1,1)$, $v_2 = (1,1,0)$ and $v_3 = (1,0,0)$ be a basis for $\mathbb{R}^3$. Find a linear transformation $T: \mathbb{R}^3 \to \mathbb{R}^2$ such that $T(v_1) = (1,0)$, $T(v_2) = (2,-1)$ and $T(v_3) = (4,3)$. **(10)** **(20)**

SECTION-B

Q. No.4. (a) Evaluate the limit: **(10)**

(i) $$\lim_{x\to\frac{\pi}{2}} (1+\cos x)^{\tan x}$$

(ii) $$\lim_{x\to 0} \left(\frac{1}{\sin x} - \frac{1}{x}\right)$$

(b) State and prove the Mean Value Theorem. **(10)** **(20)**

Q. No.5. (a) If $w = f(x^2 + y^2)$ then show that $y\left(\frac{\partial w}{\partial x}\right) - x\left(\frac{\partial w}{\partial y}\right) = 0$. **(10)**

(b) Find all the local maxima, local minima and saddle points of the given function $2x^3 + y^2 - 9x^2 - 4y + 12x - 2$. **(10)** **(20)**

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Q. No.6. (a) Evaluate the integral $\int_0^\infty x^{\frac{3}{2}}((1+2x))^{-5}\,dx$ and show that the result is $\frac{9\pi}{384}$, using **Beta** function. **(10)**

(b) Find the vertices and foci of the hyperbola **(10)** **(20)**

$$25\,x^2-16\,y^2+250x+32y+109=0.$$

SECTION-C

Q. No.7. (a) Verify that $u\,(x,y)=cosxcoshy$ is harmonic function and find a corresponding analytic function $f\,(z)=u\,(x,y)+iv\,(x,y)$. **(10)**

(b) Use Residue theorem to evaluate the integral $\int_C \frac{3\,z^2+z-1}{(z^2-1)(z-3)}\,dz$, where C is the circle $|z|=4$. **(10)** **(20)**

Q. No.8. (a) Use the Cauchy's integral formula to evaluate the integral (10) **(10)**

$$\int_C \frac{z+4}{z^2+2z+5}\,dz,\ \text{where } C \text{ is the circle } |z+1-i|=2.$$

(b) Find the three cube roots of $\sqrt{3}+i$. **(10)** **(20)**

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